## **Strain Gages and Instruments**

Tech Note TN-503

## **Measurement of Residual Stresses** by the Hole-Drilling\* Strain Gage Method

#### I. Residual Stresses and Their Measurement

Residual (locked-in) stresses in a structural material or component are those stresses that exist in the object without (and usually prior to) the application of any service or other external loads. Manufacturing processes are the most common causes of residual stress. Virtually all manufacturing and fabricating processes — casting, welding, machining, molding, heat treatment, etc. introduce residual stresses into the manufactured object. Another common cause of residual stress is in-service repair or modification. In some instances, stress may also be induced later in the life of the structure by installation or assembly procedures, by occasional overloads, by ground settlement effects on underground structures, or by dead loads which may ultimately become an integral part of the structure.

The effects of residual stress may be either beneficial or detrimental, depending upon the magnitude, sign, and distribution of the stress with respect to the loadinduced stresses. Very commonly, the residual stresses are detrimental, and there are many documented cases in which these stresses were the predominant factor contributing to fatigue and other structural failures when the service stresses were superimposed on the already present residual stresses. The particularly insidious aspect of residual stress is that its presence generally goes unrecognized until after malfunction or failure occurs.

Measurement of residual stress in opaque objects cannot be accomplished by conventional procedures for experimental stress analysis, since the strain sensor (strain gage, photoelastic coating, etc.) is totally insensitive to the history of the part, and measures only changes in strain after installation of the sensor. In order to measure residual stress with these standard sensors, the locked-in stress must be relieved in some fashion (with the sensor present) so that the sensor can register the change in strain caused by removal of the stress. This was usually done destructively in the past — by cutting and sectioning the part, by removal of successive surface layers, or by trepanning and coring.

With strain sensors judiciously placed before dissecting the part, the sensors respond to the deformation produced by relaxation of the stress with material removal. The initial residual stress can then be inferred from the measured strains by elasticity considerations. Most of these techniques are limited to laboratory applications on flat or cylindrical specimens, and are not readily adaptable to real test objects of arbitrary size and shape.

X-ray diffraction strain measurement, which does not require stress relaxation, offers a nondestructive alternative to the foregoing methods, but has its own severe limitations. Aside from the usual bulk and complexity of the equipment, which can preclude field application, the technique is limited to strain measurements in only very shallow surface layers. Although other nondestructive techniques (e.g., ultrasonic, electromagnetic) have been developed for the same purposes, these have yet to achieve wide acceptance as standardized methods of residual stress analysis.

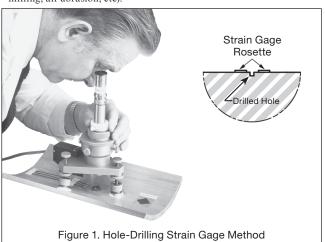
#### The Hole-Drilling Method

The most widely used modern technique for measuring residual stress is the hole-drilling strain-gage method of stress relaxation, illustrated in Figure 1.

Briefly summarized, the measurement procedure involves six basic steps:

- A special three- (or six-) element strain gage rosette is installed on the test part at the point where residual stresses are to be determined.
- The gage grids are wired and connected to a multichannel static strain indicator, such as the Vishay Micro-Measurements Model P3 (three-element gage), or System 5000 (six-element gage).

<sup>\*</sup> Drilling implies all methods of introducing the hole (i.e., drilling, milling, air abrasion, etc).



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Vishay Micro-Measurements



## Measurement of Residual Stresses by the Hole-Drilling Strain Gage Method

- A precision milling guide (Model RS-200, shown in Figure 1) is attached to the test part and accurately centered over a drilling target on the rosette.
- After zero-balancing the gage circuits, a small, shallow hole is drilled through the geometric center of the rosette.
- Readings are made of the relaxed strains, corresponding to the initial residual stress.
- Using special data-reduction relationships, the principal residual stresses and their angular orientation are calculated from the measured strains.

The foregoing procedure is relatively simple, and has been standardized in ASTM Standard Test Method E 837.1 Using commercially available equipment and supplies, and adhering to the recommendations in the ASTM standard, the hole-drilling method can be applied routinely by any qualified stress analysis technician, since no special expertise is required for making the measurements. The method is also very versatile, and can be performed in either the laboratory or the field, on test objects ranging widely in size and shape. It is often referred to as a "semidestructive" technique, since the small hole will not, in many cases, significantly impair the structural integrity of the part being tested (the hole is typically 1/32 to 3/16 in [0.8 to 4.8 mm] in both diameter and depth). With large test objects, it is sometimes feasible to remove the hole after testing is completed, by gently blending and smoothing the surface with a small hand-held grinder. This must be done very carefully, of course, to avoid introducing residual stresses in the process of grinding.

NOTE 1: In its current state of development, the hole-drilling method is intended primarily for applications in which the residual stresses are uniform throughout the drilling depth, or essentially so. While the procedures for data acquisition and reduction in such cases are well-established and straightforward, seasoned engineering judgment is generally required to verify stress uniformity and other criteria for the validity of the calculated stresses. This Tech Note contains the basic information for understanding how the method operates, but cannot, of course, encompass the full background needed for its proper application in all cases. An extensive list of technical references is provided in the Bibliography as a further aid to users of the method.

**NOTE 2:** Manual calculation of residual stresses from the measured relaxed strains can be quite burdensome, but there is available a specialized computer program, H-DRILL, that completely eliminates the computational labor.

# II. Principle and Theory of the Hole-Drilling Strain Gage Method

The introduction of a hole (even of very small diameter) into a residually stressed body relaxes the stresses at that

location. This occurs because every perpendicular to a free surface (the hole surface, in this case) is necessarily a principal axis on which the shear and normal stresses are zero. The elimination of these stresses on the hole surface changes the stress in the immediately surrounding region, causing the local strains on the surface of the test object to change correspondingly. This principle is the foundation for the hole-drilling method of residual stress measurement, first proposed by Mathar.<sup>2</sup>

In most practical applications of the method, the drilled hole is blind, with a depth which is: (a) about equal to its diameter, and (b) small compared to the thickness of the test object. Unfortunately, the blind-hole geometry is sufficiently complex that no closed-form solution is available from the theory of elasticity for direct calculation of the residual stresses from the measured strains — except by the introduction of empirical coefficients. A solution can be obtained, however, for the simpler case of a hole drilled completely through a thin plate in which the residual stress is uniformly distributed through the plate thickness. Because of this, the theoretical basis for the hole-drilling method will first be developed for the through-hole geometry, and subsequently extended for application to blind holes.

#### **Through-Hole Analysis**

Depicted in Figure 2a (following) is a local area within a thin plate which is subject to a uniform residual stress,  $\sigma_x$ . The initial stress state at any point  $P(R, \alpha)$  can be expressed in polar coordinates by:

$$\sigma_r' = \frac{\sigma_x}{2} (1 + \cos 2\alpha) \tag{1a}$$

$$\sigma_{\theta}' = \frac{\sigma_x}{2} (1 - \cos 2\alpha) \tag{1b}$$

$$\tau'_{r\theta} = -\frac{\sigma_x}{2} \sin 2\alpha \tag{1c}$$

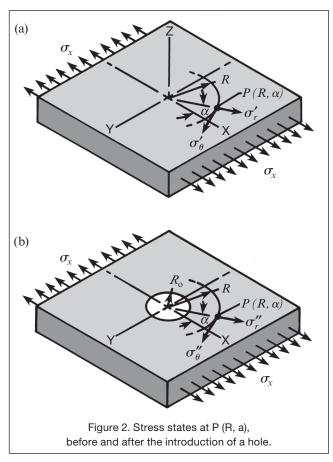
Figure 2b represents the same area of the plate after a small hole has been drilled through it. The stresses in the vicinity of the hole are now quite different, since  $\sigma_r$  and  $\tau_{r\theta}$  must be zero everywhere on the hole surface. A solution for this case was obtained by G. Kirsch in 1898, and yields the following expressions for the stresses at the point  $P(R, \alpha)$ :<sup>3</sup>

$$\sigma_r'' = \frac{\sigma_x}{2} \left( 1 - \frac{1}{r^2} \right) + \frac{\sigma_x}{2} \left( 1 + \frac{3}{r^4} - \frac{4}{r^2} \right) \cos 2\alpha$$
 (2a)

$$\sigma_{\theta}^{"} = \frac{\sigma_x}{2} \left( 1 + \frac{1}{r^2} \right) - \frac{\sigma_x}{2} \left( 1 + \frac{3}{r^4} \right) \cos 2\alpha \tag{2b}$$

$$\tau_{r\theta}^{"} = -\frac{\sigma_x}{2} \left( 1 - \frac{3}{r^4} + \frac{2}{r^2} \right) \sin 2\alpha \tag{2c}$$





where:

$$r = \frac{R}{R_o} (R \ge R_o)$$

 $R_o$  = hole radius

R = arbitrary radius from hole center

Subtracting the initial stresses from the final (after drilling) stresses gives the change in stress, or stress relaxation at point  $P(R, \alpha)$  due to drilling the hole. That is:

$$\Delta \sigma_r = \sigma_r'' - \sigma_r' \tag{3a}$$

$$\Delta \sigma_{\theta} = \sigma_{\theta}^{"} - \sigma_{\theta}^{\prime} \tag{3b}$$

$$\Delta \tau_{r\theta} = \tau_{r\theta}^{"} - \tau_{r\theta}^{'} \tag{3c}$$

Substituting Equations (1) and (2) into Equations (3) yields the full expressions for the relaxed (or relieved) stresses. If the material of the plate is homogeneous and isotropic in its mechanical properties, and linear-elastic in its stress/strain behavior, these equations can then be substituted into the biaxial Hooke's law to solve for the relieved normal strains at the point  $P(R, \alpha)$ . The resulting expressions are as follows:

$$\varepsilon_r = -\frac{\sigma_x(1+\nu)}{2E} \left[ \frac{1}{r^2} - \frac{3}{r^4} \cos 2\alpha + \frac{4}{r^2(1+\nu)} \cos 2\alpha \right]$$
 (4a)

$$\varepsilon_{\theta} = -\frac{\sigma_x(1+v)}{2E} \left[ -\frac{1}{r^2} + \frac{3}{r^4} \cos 2\alpha - \frac{4v}{r^2(1+v)} \cos 2\alpha \right]$$
 (4b)

The preceding equations can be written in a simpler form, demonstrating that along a circle at any radius  $R(R \ge R_o)$  the relieved radial and tangential strains vary in a sinusoidal manner:

$$\varepsilon_r = \sigma_x (A + B\cos 2\alpha) \tag{5a}$$

$$\varepsilon_{\theta} = \sigma_{x} \left( -A + C \cos 2\alpha \right) \tag{5b}$$

Comparison of Equations (5) with Equations (4) demonstrates that coefficients A, B, and C have the following definitions:

$$A = -\frac{1+\nu}{2E} \left(\frac{1}{r^2}\right) \tag{6a}$$

$$B = -\frac{1+v}{2E} \left[ \left( \frac{4}{1+v} \right) \frac{1}{r^2} - \frac{3}{r^4} \right]$$
 (6b)

$$C = -\frac{1+v}{2E} \left[ -\left(\frac{4v}{1+v}\right) \frac{1}{r^2} + \frac{3}{r^4} \right]$$
 (6c)

Thus, the relieved strains also vary, in a complex way, with distance from the hole surface. This variation is illustrated in Figure 3 on page 22, where the strains are plotted along the principal axes, at  $\alpha = 0^{\circ}$  and  $\alpha = 90^{\circ}$ . As shown by the figure, the relieved strains generally decrease as distance from the hole increases. Because of this, it is desirable to measure the strains close to the edge of the hole in order to maximize the strain gage output signal. On the other hand, parasitic effects also increase in the immediate vicinity of the hole. These considerations, along with practical aspects of strain gage design and application, necessitate a compromise in selecting the optimum radius (R) for gage location. Analytical and experimental studies have established a practical range of 0.3 < r < 0.45 where  $r = R_o/R$  and R is the radius to the longitudinal center of

It can be noticed from Figure 3 that for  $\alpha = 0^{\circ}$  (along the axis of the major principal stress) the relieved radial strain,  $\varepsilon_r$ , is considerably greater than the tangential strain,  $\varepsilon_{\theta}$ , in the specified region of measurement. As a result, commercial strain gage rosettes for residual stress analysis are normally designed with radially oriented grids to measure the relieved radial strain,  $\varepsilon_r$ . This being the case, only Equation  $\bigcirc$ (5a) is directly relevant for further consideration in this  $\perp$ summary. It is also evident from the figure that the relieved radial strain along the major principal axis is opposite in sign to the initial residual stress. This occurs because the



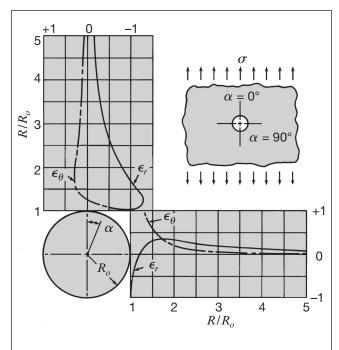


Figure 3. Variation of relieved radial and tangential strains with distance (along the principal axes) from the center of the drilled hole — uniaxial residual stress.

coefficients A and B in Equation (5a) are always negative, and (for  $\alpha = 0^{\circ}$ ) cos  $2\alpha = +1$ .

The preceding treatment considered only the simplest case, uniaxial residual stress. In practice, however, residual stresses are more often biaxial, with two nonzero principal stresses. This condition can readily be incorporated in the analysis by employing the superposition principle, which is applicable to linear-elastic material behavior. Referring to Figure 2 again, it is apparent that had the uniaxial residual stress been along only the Y axis instead of the X axis, Equations (1) and (2) would still apply, with  $\cos 2\alpha$  replaced by  $\cos 2(\alpha + 90^{\circ})$ , or by the equivalent,  $-\cos 2\alpha$ . Thus, the relieved radial strain at the point  $P(R, \alpha)$  due to uniaxial residual stress in only the Y direction can be written as:

$$\varepsilon_r^y = \sigma_v (A - B\cos 2\alpha) \tag{7}$$

And, employing the corresponding notation, Equation (5a) becomes:

$$\epsilon_r^x = \sigma_x (A + B\cos 2\alpha) \tag{8}$$

○ When both residual stresses are present simultaneously,
 ≥ the superposition principle permits algebraic addition of Equations (7) and (8), so that the general expression for the
 □ relieved radial strain due to a plane biaxial residual stress
 ○ state is:

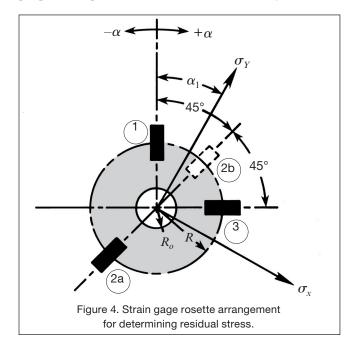
$$\varepsilon_r = \sigma_x (A + B\cos 2\alpha) + \sigma_y (A - B\cos 2\alpha)$$
 (9a)

Or, in a slightly different form,

$$\varepsilon_r = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y)\cos 2\alpha \tag{9b}$$

Equations (9) represent the basic relationship underlying the hole-drilling method of residual stress analysis. This relationship must be inverted, of course, to solve for the two principal stresses and the angle a in terms of the measured strains that accompany stress relaxation. Since there are three unknown quantities, three independent measurements of the radial strain are required for a complete solution. These three measurements can be substituted successively into Equation (9a) or Equation (9b) to yield three equations which are then solved simultaneously for the magnitudes and directions of the principal stresses.

The common procedure for measuring the relieved strains is to mount three resistance strain gages in the form of a rosette around the site of the hole before drilling. Such a rosette is shown schematically in Figure 4, where three radially oriented strain gages are located with their centers at the radius R from the center of the hole site. Although the angles between gages can be arbitrary (but must be known), a 45-degree angular increment leads to the simplest analytical expressions, and thus has become the standard for commercial residual stress rosettes. As indicated in Figure 4,  $\alpha_1$  is the acute angle from the nearer principal axis to gage no. 1, while  $\alpha_2 = \alpha_1 + 45^{\circ}$  and  $\alpha_3 = \alpha_1 + 90^\circ$ , with positive angles measured in the direction of gage numbering. It should be noted that the direction of gage numbering for the rosette type sketched in Figure 4 is clockwise, since gage no. 2, although physically at position 2a, is effectively at position 2b for gage numbering purposes. Equations (9) can be used to verify that both





locations for gage no. 2 produce the same result providing the residual stress is uniform over the area later occupied by the hole. For general-purpose applications, location 2a is usually preferred, because it minimizes the possible errors caused by any eccentricity of the drilled hole. When space for the gage is limited, as in measuring residual stresses near a weld or abutment, location 2b permits positioning the hole closest to the area of interest.

Equation (9b) can now be written three times, once for each gage in the rosette:

$$\varepsilon_1 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y)\cos 2\alpha \tag{10a}$$

$$\varepsilon_2 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos 2(\alpha + 45^\circ)$$
 (10b)

$$\varepsilon_3 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \cos 2(\alpha + 90^\circ)$$
 (10c)

When Equations (10) are solved simultaneously for the principal stresses and their direction, the results can be expressed as:

$$\sigma_{max} = \frac{\varepsilon_1 + \varepsilon_3}{4A} - \frac{1}{4B}\sqrt{(\varepsilon_3 - \varepsilon_1)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}$$
 (11a)

$$\sigma_{min} = \frac{\varepsilon_1 + \varepsilon_3}{4A} + \frac{1}{4B}\sqrt{\left(\varepsilon_3 - \varepsilon_1\right)^2 + \left(\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2\right)^2}$$
 (11b)

$$\tan 2\alpha = \frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_1 - \varepsilon_3}$$

where  $\alpha$  is the angle from the nearer principal axis to gage no. 1 (in the direction of gage numbering, if positive; or opposite, if negative).

Reversing the sense of  $\alpha$  to more conveniently define the angle from gage no. 1 to the nearer axis, while retaining the foregoing sign convention,

$$\tan 2\alpha = \frac{\varepsilon_1 - 2\varepsilon_2 + \varepsilon_3}{\varepsilon_3 - \varepsilon_1} \tag{11c}$$

The following important comments about Equations (11) should be carefully noted. These equations are very similar in appearance to the data-reduction relationships for conventional strain gage rosettes, but the differences are significant. The coefficients A and B not only incorporate the elastic properties of the test material, but also reflect the severe attenuation of the relieved strains relative to the relaxed stress. It can be observed, in addition, that the signs between terms in Equations (11a) and (11b) are opposite to those in the conventional rosette equations. This occurs because A and B are always negative; and thus, since Equation (11a) is algebraically greater than Equation (11b), the former must represent the maximum principal stress.

Equation (11c) is identical to that for a conventional three-element rectangular rosette, but must be interpreted differently to determine which principal stress is referred to gage no. 1. The following rules can be used for this purpose:

$$\varepsilon_3 > \varepsilon_1$$
:  $\alpha$  refers to  $\sigma_{max}$ 
 $\varepsilon_3 < \varepsilon_1$ :  $\alpha$  refers to  $\sigma_{min}$ 
 $\varepsilon_3 = \varepsilon_1$ :  $\alpha = \pm 45^\circ$ 
 $\varepsilon_2 < \varepsilon_1$ :  $\sigma_{max}$  at +45°
 $\varepsilon_2 > \varepsilon_1$ :  $\sigma_{max}$  at -45°

Careful consideration must also be given to determining the appropriate values for coefficients A and B. As defined algebraically in Equations (6), they apply only when the conditions imposed by the Kirsch solution are met. This solution gives the stress distribution at *points* with coordinates  $(r, \alpha)$  around a circular hole through a thin, wide plate subjected to uniform plane stress. However, comparison of Figures 3 and 4 illustrates that, since the strain gage grids in the rosette have finite areas, they sense varying strain distributions such as those plotted in Figure 3. Thus, the output of each gage tends to represent the average strain over the area of the grid. Moreover, because the grids are usually composed of parallel lines, those lines which are not directly on the centerline of a radially oriented grid are not radial. Therefore, the gages are slightly sensitive to the tangential strain, as well as the radial strain. As a result, more accurate values for the coefficients can be obtained by integrating Equations (4) over the areas of the respective gage grids. The coefficients thus determined, which account for the finite strain gage area, are designated here by  $\overline{A}$  and  $\overline{B}$  to distinguish them from the values at a point as defined by Equations (6). An alternative method for obtaining  $\overline{A}$  and  $\overline{B}$  is to measure them by experimental calibration. The procedure for doing so is described in Section III, "Determining Coefficients  $\overline{A}$  and  $\overline{B}$ ." When performed correctly, this procedure is potentially the most accurate means for evaluating the coefficients.

When employing conventional strain gage rosettes for experimental stress analysis, it is usually recommended that the strain measurements be corrected for the transverse sensitivity of the gages. Correction relationships for this purpose are given in Tech Note TN-509. These relationships are not directly applicable, however, to the relieved strains measured with a residual stress rosette by the hole-drilling method.

In the residual stress case, the individual gages in the rosette are effectively at different locations in a spatially varying strain field. As a result, the relieved axial and transverse strains applied to each gage are not related in the same manner as they are in a uniform strain field. Rigorous correction would require evaluation of the coefficient C [actually, its integrated or calibrated counterpart,  $\overline{C}$  — see Equations (6)], for both the through-hole and blind-hole geometries. Because of the foregoing, and the fact that the transverse sensitivities of Vishay Micro-Measurements residual stress rosettes are characteristically very low (approximately 1%), it is not considered necessary to correct for transverse sensitivity. Kabiri<sup>4</sup>, for example, has shown that the error due to ignoring transverse sensitivity (in the



case of uniaxial residual stress) is negligible compared to the remaining uncertainties in the measurement and datareduction procedures.

#### **Blind-Hole Analysis**

The theoretical background for the hole-drilling method was developed in the preceding treatment on the basis of a small hole drilled completely through a thin, wide, flat plate subjected to uniform plane stress. Such a configuration is far from typical of practical test objects, however, since ordinary machine parts and structural members requiring residual stress analysis may be of any size or shape, and are rarely thin or flat. Because of this, a shallow "blind" hole is used in most applications of the hole-drilling method.

The introduction of a blind hole into a field of plane stress produces a very complex local stress state, for which no exact solution is yet available from the theory of elasticity. Fortunately, however, it has been demonstrated by Rendler and Vigness<sup>5</sup> that this case closely parallels the through-hole condition in the general nature of the stress distribution. Thus, the relieved strains due to drilling the blind hole still vary sinusoidally along a circle concentric with the hole, in the manner described by Equations (9). It follows, then, that these equations, as well as the data-reduction relationships in Equations (11), are equally applicable to the blind-hole implementation of the method when appropriate blind-hole coefficients  $\overline{A}$  and  $\overline{B}$  are employed. Since these coefficients cannot be calculated directly from theoretical considerations, they must be obtained by empirical means; that is, by experimental calibration or by numerical procedures such as finite-element analysis.

Several different investigators [e.g., (20)–(23)] have published finite-element studies of blind-hole residual stress analysis. The most recently developed coefficients by Schajer are incorporated in ASTM standard E 837, and are shown graphically for the case of uniform stress in Figure 8 of this Tech Note. The computer program H-DRILL uses these coefficients.

Compared to the through-hole procedure, blind-hole analysis involves one additional independent variable; namely, the dimensionless hole depth, *Z/D* (see Figure 5). Thus, in a generalized functional form, the coefficients can be expressed as:

$$\overline{A} = f_{A}(E, v, r, Z/D)$$
 (12a)

$$\overline{B} = f_{\rm R}(E, v, r, Z/D) \tag{12b}$$

For any given initial state of residual stress, and a fixed hole diameter, the relieved strains generally increase (at a decreasing rate) as the hole depth is increased. Therefore, in order to maximize the strain signals, the hole is normally drilled to a depth corresponding to at least Z/D = 0.4 (ASTM E 837 specifies Z/D = 0.4 for the maximum hole depth).

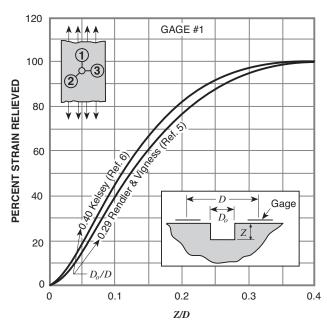


Figure 5. Relieved strain versus ratio of hole depth to gage circle diameter (strains normalized to 100% at Z/D = 0.4).

The general variation of relieved strain with depth is illustrated in Figure 5, where the strains have been normalized, in this case, to 100% at Z/D=0.4. The data include experimental results from two different investigators demonstrating the manner in which the relieved-strain function is affected by the ratio of hole diameter to gage circle diameter  $(D_o/D)$ . Both cases involve uniform uniaxial (plane) stress, in specimens that are thick compared to the maximum hole depth. The curves plotted in the figure are considered representative of the response to be expected when the residual stress is uniform throughout the hole depth.

An important contribution of the Rendler and Vigness work is the demonstration that, for any given set of material properties, E and v, coefficients  $\overline{A}$  and  $\overline{B}$  are simply geometric functions, and thus constants for all geometrically similar cases. This means that once the coefficients have been determined for a particular rosette configuration, the rosette size can be scaled upward or downward and the same coefficients will still apply when the hole diameter and depth are similarly scaled (assuming, of course, the same material). Several different approaches have been taken in attempting to remove the material dependency from  $\overline{A}$  and  $\overline{B}$ , leaving only the geometric dependence. One of these, proposed by Schajer,7 is adopted in this Tech Note. Schajer introduced two new coefficients, denoted here as  $\overline{a}$  and  $\overline{b}$ , and defined as follows:

$$\overline{a} = -\frac{2E\overline{A}}{1+v} \tag{13a}$$

$$\overline{b} = -2E\overline{B}$$

(13b)



By comparison with Equations (6), it can be seen that for the through hole, at least  $\bar{a}$  is material-independent, and  $\bar{b}$  depends only weakly on Poisson's ratio. Schajer has determined from finite-element calculations that for blind holes,  $\bar{a}$  and  $\bar{b}$  vary by less than 2% for the range of Poisson's ratio from 0.25 to 0.35.

#### III. Determining Coefficients $\overline{A}$ and $\overline{B}$

Whether the residual stress analysis application involves through-hole or blind-hole drilling, the coefficients  $\overline{A}$  and  $\overline{B}$  (or  $\overline{a}$  and  $\overline{b}$ ) must be determined to calculate the stresses from the relieved strains. In the case of the through hole, reasonably accurate values of the coefficients can be obtained for any particular case by analytical means, if desired. This is done by integrating, over the area of the gage grid, the component of strain parallel to the primary strain-sensing axis of the gage. Given the details of the grid geometry (line width and spacing, number of lines, etc.), Slightly greater accuracy may be obtained by integrating along the individual grid lines. This method cannot be applied to blind-hole analysis because closed-form expressions relating the relieved strains to the residual stress, in terms of hole depth, are not available.

#### **Experimental Calibration**

The needed coefficients for either through-hole or blind-hole analysis can always be determined by experimental calibration. This procedure is particularly attractive since it automatically accounts for the mechanical properties of the test material, strain gage rosette geometry, hole depth and diameter, and the strain-averaging effect of the strain gage grid. When performed correctly, with sufficient attention to detail, it is potentially the most accurate means for determining the coefficients. Its principal disadvantage is that the calibration must be repeated each time a different set of geometric parameters is involved.

Calibration for  $\overline{A}$  and  $\overline{B}$  is accomplished by installing a residual stress strain gage rosette on a uniaxially stressed tensile specimen made from the same material as the test part. The rosette should be oriented to align grid no. 1 parallel to the loading direction, placing grid no. 3 along the transverse axis of the specimen. Care must be taken that the tensile stress is uniform over the cross section of the test specimen; i.e., that bending stress is negligible. To minimize edge and end effects, the specimen width should be at least ten times the hole diameter, and the length between machine grips, at least five times the width. When determining  $\overline{A}$  and  $\overline{B}$  for blind-hole applications, a specimen thickness of five or more times the hole diameter is recommended. For through-hole calibration, the thickness of the calibration specimen is preferably the same as that of the test part. It is also important that the maximum applied stress during calibration not exceed one-half of the proportional limit stress for the test material. In any case, the applied stress plus the initial residual stress must be low

enough to avoid the risk of local yielding due to the stress-concentrating effect of the hole.

Basically, the calibration procedure involves measuring the rosette strains under the same applied load or calibration stress,  $\sigma_c$ , both before and after drilling the hole. Such a procedure is necessary in order to eliminate the effect of the strain relief that may occur due to the relaxation of any initial residual stress in the calibration specimen. With this technique, the observed strain difference (before and after hole drilling) is caused only by the applied calibration stress, and is uniquely related to that stress. The steps in the calibration procedure can be summarized briefly as follows, noting that the strains in only grid no. 1 and grid no. 3 need to be measured, since these grids are known to be aligned with the principal axes of the specimen.

- 1. Zero-balance the strain gage circuits.
- Apply a load, P, to the specimen to develop the desired calibration stress, σ<sub>c</sub>.
- 3. Measure strains  $\varepsilon'_1$  and  $\varepsilon'_3$  (before drilling).
- 4. Unload the specimen, and remove it from the testing machine.
- 5. Drill the hole, as described in Section V, "Experimental Techniques".
- Replace the specimen in the testing machine, re-zero the strain gage circuits, and then reapply exactly the same load, P.
- 7. Measure strains  $\varepsilon_1''$  and  $\varepsilon_3''$  (after drilling).

The calibration strains corresponding to the load, P, and the stress,  $\sigma_c$ , are then:

$$\varepsilon_{c1} = \varepsilon_1^{\prime\prime} - \varepsilon_1^{\prime}$$
$$\varepsilon_{c3} = \varepsilon_3^{\prime\prime} - \varepsilon_3^{\prime}$$

Calibration reliability can ordinarily be improved by loading the specimen incrementally and making strain measurements at each load level, both before and after drilling the hole. This permits plotting a graph of  $\sigma_c$  versus  $\varepsilon_{c1}$  and  $\varepsilon_{c3}$ , so that best-fit straight lines can be constructed through the data points to minimize the effect of random errors. It will also help identify the presence of yielding, if that should occur. The resulting relationship between the applied stress and the relieved strain is usually more representative than that obtained by a single-point determination.

Since the calibration is performed with only one nonzero principal stress, Equation (5a) can be used to develop expressions for the calibrated values of  $\overline{A}$  and  $\overline{B}$ . Successively substituting  $\alpha = 0^{\circ}$  (for grid no. 1) and  $\alpha = 90^{\circ}$  (for grid no. 3) into Equation (5a):

$$\begin{split} \varepsilon_{c1} &= \sigma_{\rm c} \left[ \overline{A} + \ \overline{B} \cos \left( 0^{\circ} \right) \right] = \sigma_{\rm c} \left( \overline{A} + \ \overline{B} \right) \\ \varepsilon_{c3} &= \sigma_{\rm c} \left[ \overline{A} + \ \overline{B} \cos \left( 2 \times 90^{\circ} \right) \right] = \sigma_{\rm c} \left( \overline{A} - \ \overline{B} \right) \end{split}$$

### Vishay Micro-Measurements



## Measurement of Residual Stresses by the Hole-Drilling Strain Gage Method

Solving for  $\overline{A}$  and  $\overline{B}$ ,

$$\bar{A} = \frac{\varepsilon_{c1} + \varepsilon_{c3}}{2\sigma_c} \tag{14a}$$

$$\overline{B} = \frac{\varepsilon_{c1} - \varepsilon_{c3}}{2\sigma_c} \tag{14b}$$

The procedure described here was applied to a throughhole specimen made from Type 304 stainless steel, and the calibration data are plotted in Figure 6. It can be seen from the figure that for this geometry ( $D_o/D = 0.35$ ) and material,  $\varepsilon_{cl}$ and  $\varepsilon_{c3}$  are  $-90\mu\varepsilon$  and  $+39\mu\varepsilon$ , respectively, when  $\sigma_c$  is 10000 psi [69 MPa]. Substituting into Equations (14),

$$\overline{A} = -0.25 \times 10^{-8} \text{ psi}^{-1} [-0.36 \times 10^{-12} \text{ Pa}^{-1}]$$
  
 $\overline{B} = -0.65 \times 10^{-8} \text{ psi}^{-1} [-0.94 \times 10^{-12} \text{ Pa}^{-1}]$ 

Although the preceding numerical example referred to the through-hole coefficients, the same procedure is followed in calibrating for full-depth blind-hole coefficients. Once  $\overline{A}$  and  $\overline{B}$  have been obtained in this manner, the corresponding material-independent coefficients,  $\overline{a}$  and  $\overline{b}$ , can be calculated from Equations (13) if the elastic modulus and Poisson's ratio of the test material are known. If desired, the procedure can then be repeated over the practical range of  $D_o/D$  to permit plotting curves of  $\overline{a}$  and  $\overline{b}$  for all cases of interest.

It should be noted that the values for the basic coefficients  $\overline{A}$  and  $\overline{B}$  obtained from a particular calibration test are strictly applicable only for residual-stress measurement conditions that exactly match the calibration conditions:

- material with the same elastic properties;
- same rosette geometry (but rosette orientation is arbitrary);
- same hole size:

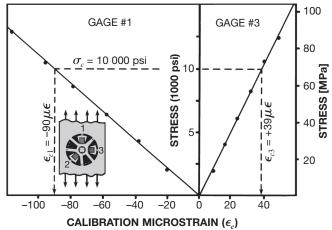


Figure 6. Stress versus relieved strain for calibration of coefficients  $\overline{A}$  and  $\overline{B}$  on 304 Stainless Steel (through-hole).

- same hole form (through hole or full-depth blind hole);
- uniform stress with depth;
- nominally uniform in-plane stress at the hole.

#### Coefficients for Vishay Micro-Measurements Residual Stress Rosettes

Vishay Micro-Measurements supplies special strain gage rosettes for residual stress analysis in four basic configurations, illustrated and described in Figure 7. Among other features, these rosette designs incorporate centering patterns for positioning the boring tool precisely at the center of the gage circle. All RE and UL designs have

#### EA-XX-062RE-120

This geometry conforms to the early Rendler and Vigness design<sup>5</sup> and has been used in most reported technical articles (see references). It is available in a range of sizes to accommodate applications requiring different hole diameters or depths.



#### CEA-XX-062UL-120

This rugged, encapsulated design incorporates all practical advantages of the CEA strain gage series (integral copper soldering tabs, conformability, etc.). Installation time and expense are greatly reduced, and



all solder tabs are on one side of the gage to simplify leadwire routing from the gage site. It is compatible with all methods of introducing the hole, and the strain gage grid geometry is identical to the 062RE pattern.

#### CEA-XX-062UM-120

Another CEA-Series strain gage, the 062UM grid widths have been reduced to facilitate positioning all three grids on one side of the measurement point as shown. With this geometry, and appropriate trimming, it is possible to position the



hole closer to welds and other irregularities. The user should be reminded, however, that the data reduction equations are theoretically valid only when the holes are well removed from free boundaries, discontinuities, abrupt geometric changes, etc. The UM design is compatible with all methods of introducing the hole.

#### N2K-XX-030RR-350/DP

The K-alloy grids of this open-faced sixelement rosette are mounted on a thin, high-performance laminated polyimide film backing. Solder tabs are duplex copper plated for ease in making solder joints for lead attachment. Diametrically opposed circumferential and radial grids are to be wired in half-bridge configurations.

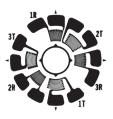


Figure 7. Residual stress strain gage rosettes (shown approximately 2X).



geometrically similar grid configurations, with the gagecircle diameter equal to 3.25 times the active gage length. The 062RE rosette, for example, has a gage-circle diameter of 0.202 in [5.13 mm]. Because of this similitude, the same material-independent coefficients  $\bar{a}$  and  $\bar{b}$  apply to all sizes of the RE rosette, and to the UL rosette, for geometrically similar holes (i.e., for the same Do/D and Z/D ratios). The 062UM rosette configuration has the same ratio of gage circle to grid length, but the grids are narrower to permit their close grouping on one side of the hole. As a result, the sensitivity of the gage to the relieved strains is slightly greater, and coefficients specific to the 062UM are required for data reduction.

The 030RR rosette is fundamentally different from the other rosettes illustrated in Figure 7. To begin with, this rosette includes both radially and circumferentially oriented grids which are to be connected as half-bridge pairs. The 030RR rosette incorporates a number of features that contribute to its greater output and higher accuracy compared to conventional three-element rosettes: (a) the individual gridlines in the radial elements are purely radial, instead of being simply parallel to the central gridline as in the other rosettes; (b) for a given maximum hole diameter, the outermost radius of the grids is considerably less than for the corresponding conventional rosettes, and thus the grids sense slightly greater average released strains; and (c) since the radial and circumferential grids get connected in a half-bridge configuration, the bridge output is augmented correspondingly, and the circuit is intrinsically selftemperature compensating. As a result of these features, the 030RR rosette yields about twice the output of the three-element rosettes for a given state of residual stress, while displaying better stability and accuracy.

Since the sign of the residual stress is of primary importance in determining its effect on the structural integrity of any mechanical component, the user of the six-element rosette (030RR) must exercise care in connecting the rosette grids into Wheatstone bridge circuits. To obtain the correct sign in the instrument output signal, the radially oriented grids should always be connected between the positive excitation and the negative signal terminals, while the tangentially oriented grids are to be connected between the negative signal and negative excitation terminals.

The  $\overline{a}$  and  $\overline{b}$  coefficients for Vishay Micro-Measurements residual stress rosettes are provided graphically in Figure 8 on page 28, where the solid lines apply to full-depth blind holes and the dashed lines to through holes assuming, in both cases, that the initial residual stress is uniform with depth. Both the through-hole and full-depth blind-hole coefficients plotted in Figure 8 were determined by a combination of finite-element analysis and experimental verification. These coefficients are also supplied numerically in tabular form in ASTM E 837-99, where RE/UL rosettes are designated as Type A, UM rosettes as Type B, and RR rosettes as Type C. For the blind-hole coefficients in the

ASTM standard, "full depth" corresponds to a value of 0.40 for the depth to rosette-mean-diameter-ratio, Z/D.

#### IV. Measuring Nonuniform Residual Stresses

The coefficients given in this Tech Note and in ASTM E 837-99 are strictly applicable only to situations in which the residual stresses do not vary in magnitude or direction with depth from the test-part surface. In reality, however, residual stresses may often vary significantly with depth, due, for example, to different manufacturing processes such as casting, forging, heat treatment, shot peening, grinding, etc.

Numerous finite-element studies have been made in attempts to treat this situation [see, for instance, references (20) through (23)]. The results of the finite element work by Schajer have been incorporated in a computer program, H-DRILL, for handling stress variation with depth. When the measured strains from hole drilling do not fit the reference curves in Figure 10, or when there is any other basis for suspecting significant nonuniformity, the program H-DRILL or some other finite-element-based program is necessary to accurately determine the stresses from the measured relaxed strains.

#### V. Experimental Techniques

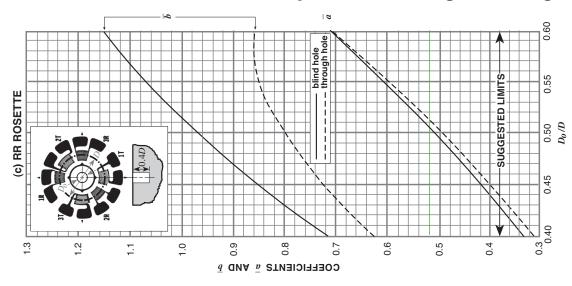
As in all experimental methods, proper materials, application procedures, and instrumentation are essential if accurate results are to be obtained. The accuracy of the hole-drilling method is dependent chiefly upon the following technique-related factors:

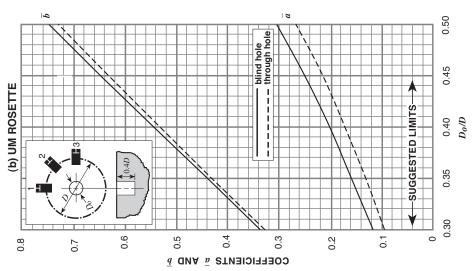
- strain gage selection and installation.
- hole alignment and boring.
- strain-indicating instrumentation.
- understanding the mechanical properties of the test material.

#### **Strain Gage Selection and Installation**

Installing three individual strain gages, accurately spaced and oriented on a small circle, is neither easy to do nor advisable, since small errors in gage location or orientation can produce large errors in calculated residual stresses. The rosette configurations shown in Figure 7 have been designed and developed by Vishay Micro-Measurements specifically for residual stress measurement. The rosette designs incorporate centering marks for aligning the boring tool precisely at the center of the gage circle, since this is critical to the accuracy of the method. $^{9,10,11}$  All configurations are available in a range of temperature compensations for use on common structural metals. However, only the RE design is offered in different sizes \_ (031RE, 062RE, and 125RE), where the three-digit prefix represents the gage length in mils (0.001 in [0.0254 mm]).  $\geq$ The RE design is available either open-faced or with Option SE (solder dots and encapsulation).







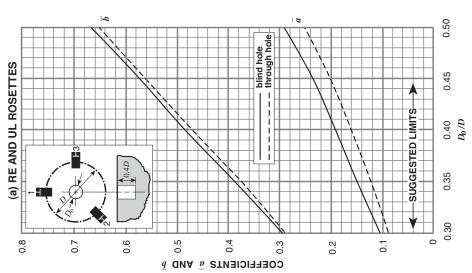


Figure 8. Full-depth data-reduction coefficients  $\overline{a}$  and  $\overline{b}$  versus dimensionless hole diameter (typical) for Vishay Micro-Measurements residual stress rosettes, in accordance with ASTM E 837.



The UL and UM configurations are supplied in ½6 in [1.6 mm] gage length, and both types are fully encapsulated. Both configurations have integral, coppercoated solder tabs, and offer all advantages of the popular C-Feature strain gage series. These residual stress rosettes are constructed with self-temperature-compensated constantan foil, mounted on a flexible polyimide carrier. Gage resistance is 120 ohms ±0.4%. The 030RR six-element rosette incorporates self-temperature-compensated K-alloy (modified Karma) foil on a laminated polyimide film backing. Solder tabs are duplex copper plated for ease in making solder connections. Gage resistance is 350 ohms ±0.4%.

Surface preparation for installing the rosettes is basically standard, as described in Application Note B-129. Caution should be observed, however, in abrading the surface of the test object, since abrasion can alter the initial state of residual stress.<sup>12</sup> In general, it is important that all surface-preparation and gage-installation procedures be of the highest quality, to permit accurate measurement of the small strains typically registered with the hole-drilling method. As evidenced by the calibration data in Figure 6, the relieved strains corresponding to a given residual stress magnitude are considerably lower than those obtained in a conventional mechanical test at the same stress level. Because of the small measured strains, any drift or inaccuracy in the indicated gage output, whether due to improper gage installation, unstable instrumentation, or otherwise, can seriously affect the calculated residual stresses.

#### **Strain-Measurement Instrumentation**

The residual stress rosettes described in Figure 7 impose no special instrument requirements. When measurements are to be made in the field, a portable, battery-operated static strain indicator, supplemented by a precision switch-andbalance unit, is ordinarily the most effective and convenient instrumentation. The Model P3 Strain Indicator and Recorder is ideally suited for this type of application. In the laboratory it may be convenient to use a computerized automatic data system such as System 5000, which will rapidly acquire and record the data in an organized, readily accessible form. A special offline, Windows®based computer program H-DRILL is also available to perform the calculations and determine the residual stress magnitudes in accordance with ASTM E 837. The database for the program includes values of the coefficients  $\overline{a}$  and  $\overline{b}$ for blind holes, and covers the full range of recommended hole dimensions applicable to all Vishay Micro-Measurements residual stress rosettes.

### Alignment

Rendler and Vigness observed that "the accuracy of the (hole-drilling) method for field applications will

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be directly related to the operator's ability to position the milling cutter precisely in the center of the strain gage rosette." More recent studies have quantified the error in calculated stress due to eccentricity of the hole. For example, with a hole that is 0.001 in [0.025 mm] off-center of the 062RE or 062UL rosette, the error in calculated stress does not exceed 3% (for a uniaxial stress state). 9,10,11 In practice, the required alignment precision to within 0.001 in [0.025 mm] is accomplished using the RS-200 Milling Guide shown in Figure 9. The milling guide is normally secured to the test object by bonding its three foot pads with a quick-setting, frangible adhesive. A microscope is then installed in the unit and visual alignment is achieved with the aid of the four X-Y adjustment screws on the exterior of the guide.

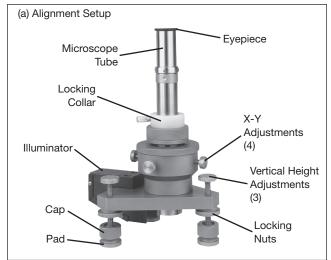
#### **Boring**

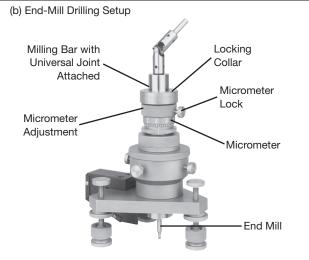
Numerous studies on the effects of hole size and shape and machining procedures have been published. Rendler and Vigness<sup>5</sup> specified a specially dressed end mill which is compatible with the residual stress rosettes of Figure 7. The end mill is ground to remove the side cutting edges, and then relieved immediately behind the cutting face to avoid rubbing on the hole surface. It is imperative that the milling cutter be rigidly guided during the drilling operation so that the cutter progresses in a straight line, without side pressure on the hole, or friction at the noncutting edge. These end mills generate the desired flat-bottomed and squarecornered hole shape at initial surface contact, and maintain the appropriate shape until the hole is completed. In doing so, they fulfill the incremental drilling requirements as stipulated in ASTM E 837. Specially dressed end mills offer a direct and simple approach when measuring residual stresses on readily machinable materials such as mild steel and some aluminum alloys. Figure 9b shows the RS-200 Milling Guide with the microscope removed and the endmill assembly in place. The end mill is driven through the universal joint at the top of the assembly, by either a hand drill or variable-speed electric drill.

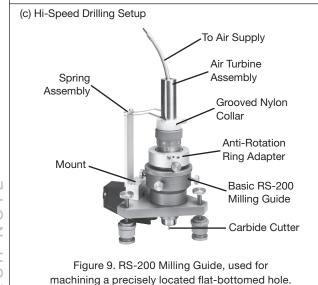
In 1982, Flaman<sup>13</sup> first reported excellent results for residual stress measurement using a high-speed (up to 400 000 rpm) air turbine and carbide cutters. This technique maintains all of the advantages (good hole shape, adaptability to incremental drilling, etc.) of the specially dressed end mill while providing for easier operation and more consistent results. Further, the air turbine is highly recommended for use with test materials that are difficult to machine, such as Type 304 stainless steel. Carbide cutters are not effective for penetrating glass, most ceramics, very hard metals, etc.; but diamond cutters have shown promise on these kinds of test materials. Figure 9c shows the air turbine/carbide cutter assembly installed in the same basic RS-200 Milling Guide.

Bush and Kromer<sup>14</sup> reported, in 1972, that stress-free holes are achieved using abrasive jet machining (AJM). Modifi-









cations and improvements were made to AJM by Procter

and Beaney, <sup>10</sup> and by Bynum. <sup>15</sup> Wnuk <sup>16</sup> experienced good results by mechanically adapting AJM apparatus for use in the RS-200 Milling Guide. The principal advantage of AJM is its reported ability to generate stress-free holes in virtually all materials. Its chief limitations center about the considerable changes in hole shape as a function of hole depth. The initial shape is saucer-like, and the final is cylindrical with slightly rounded corners. During drilling there is also uncertainty as to the actual hole depth at any stage. These factors make AJM a less practical technique for determining the variation of relieved strain with hole depth, as recommended in ASTM E 837.

#### **Mechanical Properties**

As in any form of experimental stress analysis, the accuracy of residual stress measurement is limited by the accuracies to which the elastic modulus and Poisson's ratio are known. But typical uncertainties in the mechanical properties of common steel and aluminum alloys are in the neighborhood of 1 to 3% and, as such, are minor contributors to potential errors in residual stress analysis. Much larger errors can be introduced by deviations from the assumptions involved in the basic theory, as described in Section II. A key assumption, for instance, is linear-elastic material behavior. If the stress/strain relationship for the test material is nonlinear (as is the case for cast iron), due to yielding or other causes, the calculated residual stresses will be in error.

When the initial residual stress is close to the yield strength of the test material, the stress concentration caused by the presence of the hole may induce localized yielding. It is therefore necessary to establish a threshold level of residual stress below which yielding is negligible. This problem has been studied both experimentally and analytically, and there is substantial agreement among the different investigations. 10,17,18 That is, errors are negligible when the residual stress is less than 70% of the proportional limit of the test material — for both blind holes and through holes. On the other hand, when the initial residual stress is equal to the proportional limit, errors of 10 to 30% (and greater) have been observed. The error magnitude obviously depends on the slope of the stress/strain diagram in the post-yield region; and tends to increase as the curve becomes flatter, approaching the idealized perfectly plastic behavior.18

# VI. Data Reduction and Interpretation — Blind Hole

As recommended in ASTM E 837, it is always preferable to drill the hole in small increments of depth, recording the observed strains and measured hole depth at each increment. This is done to obtain data for judging whether the residual stress is essentially uniform with depth, thus validating the use of the standard full-depth coefficients  $\bar{a}$  and  $\bar{b}$  for calculating the stress magnitudes. If incremental measurements are not taken, there is no means for making



inferences about stress uniformity, and the calculated residual stress may be considerably in error. In such cases, when the stress varies with depth, it should be realized that the calculated stress is always lower than the actual maximum.

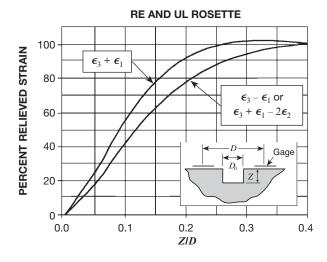
There is currently no absolute criterion for verifying stress uniformity from the surface of the test piece to the bottom of a full-depth hole. However, the incremental data, consisting of relieved strain versus hole depth, can be used in two different ways to aid in detecting a nonuniform stress distribution. The first of these is to calculate, for each depth increment, the sums and differences of the measured strain data,  $\varepsilon_3 + \varepsilon_1$  and  $\varepsilon_3 - \varepsilon_1$  respectively. Express each set of data as fractions of their values when the hole depth equals 0.4 times the mean diameter of the strain gage circle. Plot these percent strains versus normalized hole depth. These graphs should yield data points very close to the curves shown in Figure 10. Data points which are removed from the curves in Figure 10 indicate either substantial stress nonuniformity or strain measurement errors. In either case, the measured data are not acceptable for residual stress calculations using the full-depth coefficients  $\overline{a}$  and  $\overline{b}$ .

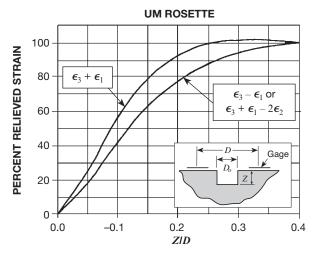
When a principal residual stress direction is closer to the axial direction of gage no. 2 in Figure 4 than to either gage nos. 1 or 3, the strain sum  $\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2$  will be numerically larger than  $\varepsilon_3 - \varepsilon_1$ . In such a case, the percent strain data check should be done using  $\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2$  instead of  $\varepsilon_3 - \varepsilon_1$ . NOTE: This graphical test is not a sensitive indicator

NOTE: This graphical test is not a sensitive indicator of stress field uniformity. Specimens with significantly nonuniform stress fields can yield percentage relieved strain curves substantially similar to those shown in Figure 10. The main purpose of the test is to identify grossly nonuniform stress fields. Further, the graphical comparison test using  $\varepsilon_3 - \varepsilon_1$  or  $\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2$ , for example, becomes ineffective when the residual stress field approaches equal biaxial tension or compression ( $\varepsilon_1 \cong \varepsilon_2 \cong \varepsilon_3$ ) as expected in surface blasting and heat treating procedures. Comparison to the  $\varepsilon_3 + \varepsilon_1$  plot is ineffective when  $\varepsilon_3 = -\varepsilon_1$  (pure shear); however, this condition is relatively uncommon in the practical industrial setting.

#### **Limitations and Cautions**

Finite-element studies of the hole-drilling method by Schajer and by subsequent investigators<sup>20,21,22,23</sup> have shown that the change in strain produced in drilling through any depth increment (beyond the first) is caused only partly by the residual stress in that increment. The remainder of the incremental relieved strain is generated by the residual stresses in the preceding increments, due to the increasing compliance of the material, and the changing stress distribution, as the hole is deepened. Moreover, the relative contribution of the stress in a particular increment to the corresponding incremental change in strain decreases rapidly with distance from the surface. As a result, the





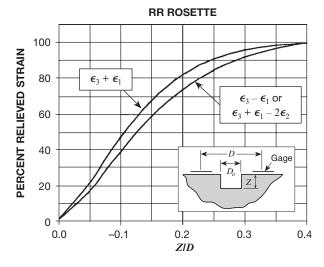


Figure 10. Percent strain versus normalized hole depth for uniform stress with depth for different rosette types, after ASTM E 837.1



total relieved strain at full-hole depth is predominantly influenced by the stresses in the layers of material closest to the surface — say, in the upper third, or perhaps half, of the hole depth. At hole depths corresponding to Z/D > 0.2, the stresses in these increments have very little effect on the observed strains. This behavior is confirmed (for uniform stress) by the shape of the normalized strain graph in Figure 5, where about 80% of the total strain relief normally occurs in the first half of the hole depth. Because of these characteristics, little, if any, quantitative interpretation can safely be made of the incremental strain data for increments beyond Z/D = 0.2, irrespective of the analytical method employed for data reduction.

To summarize, the ideal application of the hole-drilling method is one in which the stress is essentially uniform with depth. For this case, the data-reduction coefficients are well-established, and the calculated stresses sufficiently accurate for most engineering purposes — assuming freedom from significant experimental errors. Incremental drilling and data analysis should always be performed, however, to verify the stress uniformity. If the graph of percent-strain-relieved versus Z/D (see Figure 10) suggests that the stress is nonuniform with hole depth, then the procedure specified by ASTM E 837 is not applicable, and a program such as H-DRILL must be used to calculate the

Error and uncertainty are always present, in varying degrees, in all measurements of physical variables. And, as a rule, their magnitudes are strongly dependent on the quality of the experimental technique as well as the number of parameters involved. Since residual stress determination by the hole-drilling method involves a greater number and variety of techniques and parameters than routine experimental stress analysis, the potential for error is correspondingly greater. Because of this, and other considerations briefly outlined in the following, residual stresses cannot usually be determined with the same accuracy as stresses due to externally applied static loads.

Introduction of the small hole into the test specimen is one of the most critical operations in the procedure. The instruction manual for the RS-200 Milling Guide contains detailed directions for making the hole; and these should be followed rigorously to obtain maximum accuracy. The hole should be concentric with the drilling target on the special strain gage rosette. It should also have the prescribed shape in terms of cylindricity, flat bottom, and sharp corner at the surface. It is particularly necessary that the requirements on hole configuration be wellsatisfied when doing incremental drilling to examine stress variation with depth. Under these same circumstances, it is important that the hole depth at each drilling increment be measured as accurately as possible, since a small absolute error in the depth can produce a large relative error in the calculated stress. Because of practical limitations on measuring shallow hole depths, the first depth increment should ordinarily be at least 0.005 in [0.13 mm]. Accurate measurement of the hole diameter is also necessary. Finally, it is imperative that the hole be drilled (milled) without introducing significant additional residual stresses. To the degree that any of the foregoing requirements fail to be met, accuracy will be sacrificed accordingly.

Strains relieved by drilling the hole are measured conventionally, with static strain instrumentation. The indicated strains are characteristically much smaller, however, than they would be for the same stress state in an externally loaded test part. As a result, the need for stable, accurate strain measurement is greater than usual. With incremental drilling, the strains measured in the first few depth increments can be especially low, and errors of a few microstrain can cause large percentage errors in the calculated stresses for those depths.

Beyond the above, it is also necessary that the underlying theoretical assumptions of the hole-drilling method be reasonably satisfied. In full-depth drilling per ASTM E 837, the stress must be essentially uniform with depth, both in magnitude and direction, to obtain accurate results. With finite-element and other procedures for investigating stress variation in subsurface layers, it is required only that the directions of the principal stresses not change appreciably with depth. As for all conventional strain-gage rosette measurements, the data-reduction relationships assume that the stress is uniformly distributed in the plane of the test surface. However, for residual stress measurements the effective "gage-length" is the hole diameter rather than the relatively large dimensions of the overall rosette geometry. Consequently, uncertainties introduced by in-plane surface strain gradients are generally lower for residual stress determination than for conventional static load testing. No generalization can currently be made about the effects of steeply varying, nonlinear stress distributions in subsurface planes parallel to the rosette.

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